ADVANCEMENTS IN STATNAMIC DATA REGRESSION TECHNIQUES

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ABSTRACT

Until recently, the analysis of Statnamic test data has typically incorporated the "Unloading Point Method" (Middendorp et. al, 1992) to determine an equivalent static capacity. The UP method requires that the foundation move as a rigid body, thus excluding stress wave phenomenon from the analysis. If this requirement is met the foundation capacity can be determined using this simplified method. However, many foundations do not meet the UP criteria (e.g. fixed end or relatively long piles) and have proven difficult to analyze without more complex techniques. This paper presents a new analysis method that uses measured strain data as well as the standard Statnamic test data to determine foundation capacity. This new method discretizes the foundation into smaller segments that each meet the rigid body criteria of the UP method. Thereby, a more refined inertia and viscous damping evaluation can be implemented that individually determines the contributions from the various segments. This approach, termed the "Segmental Unloading Point" (SUP) method, is developed herein and then demonstrated with results from full-scale Statnamic test data.

INTRODUCTION

Since its inception in1988, Statnamic testing of deep foundations has gained popularity with many designers largely due to its time efficiency, cost effectiveness, data quality, and flexibility in testing existing foundations. Where large capacity static tests may take up to a week to set up and conduct, the largest of Statnamic tests typically takes no more than a few days. Further, multiple smaller-capacity tests can easily be completed within a day. The direct benefit of this time efficiency is the cost savings to the client and the ability to conduct more tests within a given budget. Additionally, this test method has boosted quality assurance by giving the contractor the ability to test foundations thought to have been compromised by construction difficulties without significantly affecting production.

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Statnamic testing is designated as a rapid load test that uses the inertia of a relatively small reaction mass instead of a reaction structure to produce large forces. Rapid load tests are differentiated from static and dynamic load tests by comparing the duration of the loading event with respect to the axial natural period of the foundation (2L/C). Test durations longer than 1000 L/C are considered static loadings and those shorter than 10 L/C are consider dynamic, where L represents the foundation length and C represents the strain wave velocity (Janes et al., 2000; Kusakabe et.al, 2000). Tests with a duration between 10L/C and 1000 L/C are denoted as rapid load tests. The duration of the Statnamic test is typically 100 to 120 milliseconds, but is dependant on the ratio of the applied force to the weight of the reaction mass. Longer duration tests of up to 500 milliseconds are possible but require a larger reaction mass.

The Statnamic force is produced by quickly-formed high pressure gases that in turn launch a reaction mass upward at up to twenty times the acceleration of gravity. The equal and opposite force exerted on the foundation is simply the product of the mass and acceleration of the reaction mass. It should be noted that the acceleration of the reaction mass is not significant in the analysis of the foundation; it is simply a by-product of the test. Secondly, the load produced is not an impact in that the mass is in contact prior to the test. Further, the test is over long before the masses reach the top of their flight. The parameters of interest are only those associated with the movement of the foundation (i.e. force, displacement, and acceleration).

Typical analysis of Statnamic data relies on measured values of force, displacement and acceleration. A soil model is not required, hence, the results are not highly user dependent. A new method of analysis is introduced that extends present methods by incorporating additional measured values of strain at discrete points along the length of the foundation. In the ensuing sections a discussion of analysis methods and their applicability will be presented. Full details on the development of this method can be found elsewhere (Lewis, 1999).

PRESENT ANALYSIS PROCEDURES

The Statnamic forcing event induces foundation motion in a relatively short period of time and hence acceleration and velocities will be present. The accelerations are typically small (1-2 g's), however the enormous mass of the foundation when accelerated resists movement due to inertia and as such the fundamental equation of motion applies, Equation 1.

$$F = ma + cv + kx \qquad \text{Equation 1}$$

where,
F = forcing event
m = mass of the foundation
a = acceleration of the displacing body
v = velocity of the displacing body
c = viscous damping coefficient
k = spring constant of the displacing system
x = displacement of the body

The equation of motion is generally described using four terms: forcing, inertial, viscous damping, and stiffness. The forcing term (F) denotes the load application which varies with time and is equated to the sum of remaining three terms. The inertial term (ma) is the force which is generated from the tendency of a body to resist motion, or to keep moving once it is set in motion (Young,1992). The viscous damping term (cv) is best described as the velocity dependant resistance to movement. The final term (kx), represents the classic system stiffness, which is the static soil resistance.

When this equation is applied to a pile/soil system the terms can be redefined to more accurately describe the system. This is done by including both measured and calculated terms. The revised equation is displayed below:

$$F_{Statnamic} = (ma)_{Foundation} + (cv)_{Foundation} + F_{Static} \qquad \text{Equation 2}$$

where, $F_{Statnamic}$ is the measured Statnamic force, m is the calculated mass of the foundation, a is the measured acceleration of the foundation, c is the viscous damping coefficient , v is the calculated velocity, and F_{static} is the derived pile/soil static response.

There are two unknowns in the revised equation F_{static} and c, thus the equation is under specified. F_{static} is the desired value, so the variable c must be obtained to solve the equation. Middendorp (1992) presented a method to calculate the damping coefficient referred to as the Unloading Point Method (UP). With the value of c known, the static force can be calculated. This force, termed "Derived Static," represents an equivalent soil response to that produced by a traditional static load test.

UP DESCRIPTION

The UP is a simple method which allows the equivalent static resistance to be derived from the measured Statnamic quantities. It uses a simple single degree of freedom model to represent the foundation/soil system as a rigid body supported by a non-linear spring and a linear dashpot in parallel (see Figure 1). The spring represents the static soil response (F_{Static}) which includes the elastic response of the foundation as well as the foundation/soil interface and surrounding soil response. The dashpot is used to represent the dynamic resistance which depends on the rate of pile penetration (Nishimura, 1995).

The UP makes two primary assumptions in its determination of "c." The first is the static capacity of the pile is constant when it plunges as a rigid body. The second is that the damping coefficient is constant throughout the test. By doing so a window is defined in which to calculate the damping coefficient. The first point of interest (1) is that of maximum Statnamic Force. At this point the static resistance is assumed to have become steady state, for the purpose of calculating "c". Thus, any extra resistance is attributed to that of the dynamic forces (ma and cv). The next point of interest (2) is that of zero velocity which has been termed the "Unloading Point." Figure 2 shows a typical Statnamic load-displacement curve which denotes points (1) and (2). At this point the foundation is no longer moving and the resistance due to damping is zero. The static resistance, used to calculate "c" from point (1) to (2), can then be calculated by the following equation:

$$F_{Static_{UP}} = F_{Statnamic} - (ma)_{Foundation} \qquad \text{Equation 3}$$

where, $F_{Statnamic}$, m, and a are all known parameters; $F_{Static\,UP}$ is the static force calculated at (2) and assumed constant from (1) to (2).

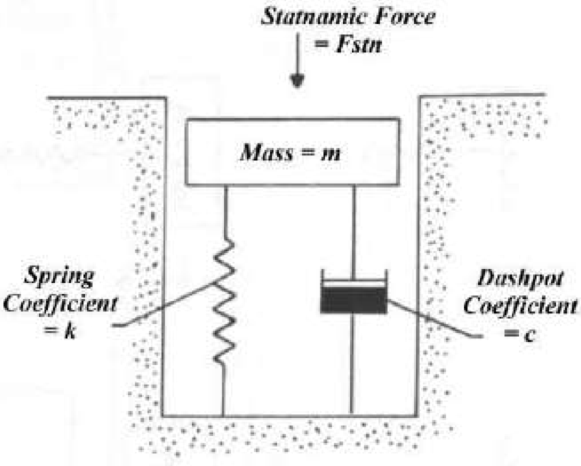

Figure 1 Single D.O.F. Model (After Das 1994)

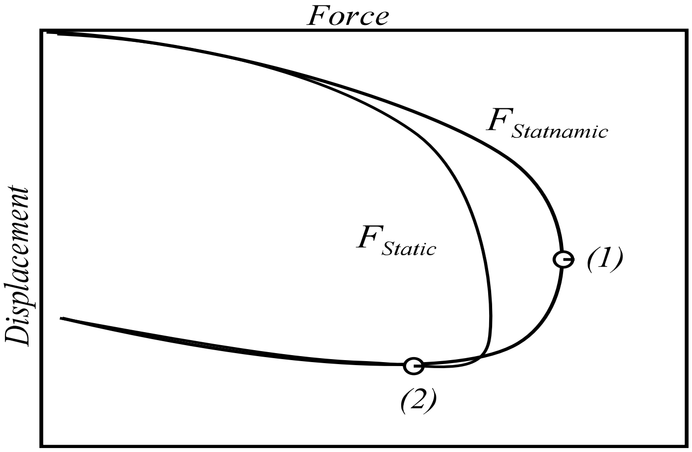

Figure 2 UP time window for C determination.

Next, the damping coefficient can be calculated throughout this range, from maximum force (1) to zero velocity (2). The following equation is used to calculate *c*:

$$c = \frac{F_{Statnamic} - F_{Static_{UP}} - (ma)_{Foundation}}{v_{Foundation}} \qquad \text{Equation 4}$$

Damping values over this range should be fairly constant. Often the average value is taken as the damping constant, but if a constant value occurs over a long period of time it should be used (see Figure 3). Note that as v approaches zero at point (2), values of *c* can be different from that of the most representative value and therefore the entire trend should be reviewed. Finally the derived static response can be calculated as follows:

$$F_{Static} = F_{Statnamic} - (ma)_{Foundation} - (cv)_{Foundation} \qquad \text{Equation 5}$$

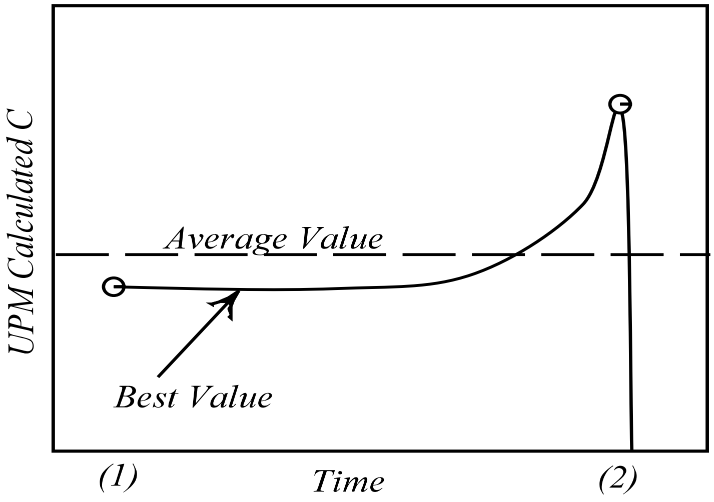

Figure 3 Variation in *C* between times (1) and (2).

Currently software is available to the public that can be used in conjunction with Statnamic test data to calculate the derived static pile capacity using the UP Method (Garbin, 1999). This software was developed by the University of South Florida and the Federal Highway Administration and can be downloaded from www.eng.usf.edu/~gmullins under the Statnamic Analysis Workbook (SAW™) heading.

UP SHORTCOMINGS

The UP has proven to be a valuable tool in predicting damping values when the foundation acts as a rigid body. However, as the pile length increases an appreciable delay can be introduced between the movement of the pile top and toe, hence negating the rigid body assumption. This occurrence also becomes prevalent when an end bearing condition exists; in this case the lower portion of the foundation is prevented from moving jointly with the top of the foundation.

Middendorp (1995) defines the "Wave Number" (N_w) to quantify the applicability of the UP. The wave number is calculated by dividing the wave length (D) by the foundation depth (L). D is obtained by multiplying the wave speed c in length per second by the load duration (T) in seconds. Thus, the wave number is calculated by the following equation:

$$N_w = \frac{D}{L} = \frac{cT}{L} \qquad \text{Equation 6}$$

Through empirical studies Middendorp determined that the UP would accurately predict static capacity, from Statnamic data, if the wave number is greater than 12. Nishimura (1995) established a similar threashold at a wave number of 10. Using wave speeds of 5000 m/s and 4000m/s for steel and concrete respectively and a typical Statnamic load duration, the UP is limited to piles shorter than 50 m (steel) and 40 m (concrete). Wave number analysis can be used to determine if stress waves will develop in the pile. However, this does not necessarily satisfy the rigid body requirement of the UP.

Statnamic tests cannot always produce wave numbers greater than 10, and as such there have been several methods suggested to accommodate stress wave phenomena in Statnamically tested long piles (Middendorp, 1995). Due to limitations on paper length these methods are not presented.

MODIFIED UNLOADING POINT METHOD

Given the shortcomings of the UP, users of Statnamic testing have developed a remedy for the problematic condition that arises most commonly. The scenario involves relatively short piles (N_w>10) that do not exhibit rigid body motion, but rather elastically shorten within the same magnitude as the permanent set. This is typical of rock-socketed drilled shafts or piles driven to dense bearing strata that are not fully mobilized during testing. The consequence is that the top of pile response (i.e. acceleration, velocity, and displacement) is significantly different from that of the toe. The most drastic subset of these test results show zero movement at the toe while the top of pile elastically displaces in excess of the surficial yield limit (e.g. upwards of 25 mm). Whereas with plunging piles (rigid body motion) the difference in movement (top to toe) is minimal and the average acceleration is essentially the same as the top of pile acceleration; tip restrained piles will exhibit an inertial term that is twice as large when using top of pile movement measurements to represent the entire pile.

The Modified Unloading Point Method (MUP), developed by Justason (1997), makes use of an additional toe accelerometer that measures the toe response. The entire pile is still assumed to be a single mass, m, but the acceleration of the mass is now defined by the average of the top and toe movements. A standard UP is then conducted using the applied top of pile Statnamic force and the average accelerations and velocities. The derived static force is then plotted against top of pile displacement as before. This simple extension of the UP has successfully overcome most problematic data sets. Plunging piles instrumented with both top and toe accelerometers have shown little analytical difference between the UP and the MUP. However, MUP analyses are now recommended whenever both top and toe information is available.

NEED FOR ADVANCEMENT

Although the MUP provided a more refined approach to some of the problems associated with UP conditions, there still exists a scenario where it is difficult to interpret Statnamic data with present methods. This is when the wave number is less than ten (relatively long piles). In these cases the pile may still only experience compression (no tension waves) but the delay between top and toe movements causes a phase lag. Hence an average of top and toe movements does not adequately represent the pile.

SUP METHOD DESCRIPTION AND GENERAL PROCEDURE

The fundamental concept of the SUP is that the acceleration, velocity, displacement, and force on each segment can be determined using strain gage measurements along the length of the pile. Individual pile segment displacements are determined using the relative displacement as calculated from strain gage measurements and an upper or lower measured displacement. The velocity and acceleration of each segment are then determined by numerically differentiating displacement then velocity with respect to time. The segmental forces are determined by calculating the difference in force from two strain gage levels.

Typically the maximum number of segments is dependent on the available number of strain gage layers. However, strain gage placement does not necessitate assignment of segmental boundaries; as long as the wave number of a given segment is greater than 10, the segment can include several strain gage levels within its boundaries. The number and the elevation of strain gage levels are usually determined based on soil stratification; as such, it can be useful to conduct an individual segmental analysis to produce the shear strength parameters for each soil strata. A reasonable upper limit on the number of segments should be adopted because of the large number of mathematical computations required to complete each analysis. Figure 4 is a sketch of the SUP pile discretization.

The notation used for the general SUP case defines the pile as having m levels of strain gages and $m+1$ segments. Strain gage locations are labeled using positive integers starting from 1 and continuing through m. The first gage level below the top of the foundation is denoted as GL^1 where the superscript defines the gage level. Although there are no strain gages at the top of foundation, this elevation is denoted as GL^0. Segments are numbered using positive integers from 1 to $m+1$, where segment 1 is bounded by the top of foundation (GL^0) and GL^1. Any general segment is denoted as segment n and lies between GL^{n-1} and GL^n. Finally, the bottom segment is denoted as segment $m+1$ and lies between GL^m and the foundation toe.

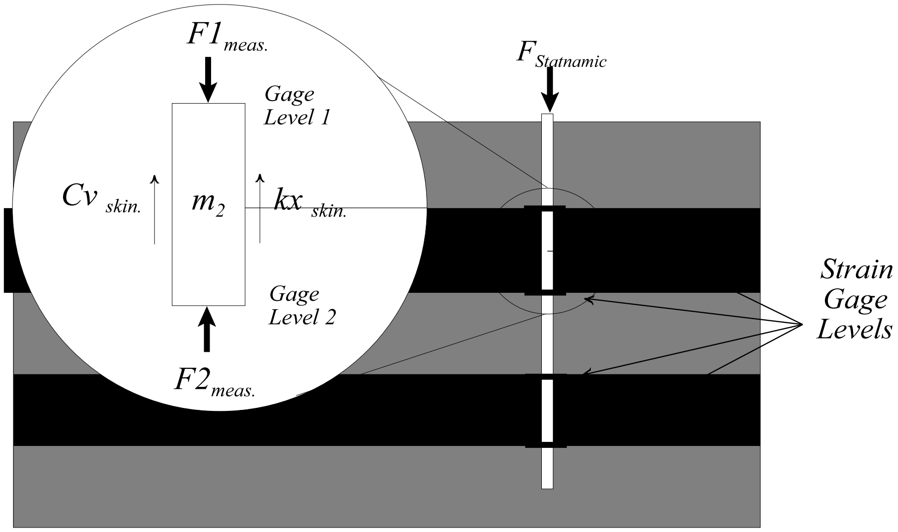

Figure 4 Segmental Free Body Diagram

CALCULATION OF SEGMENTAL MOTION PARAMETERS

The SUP analysis defines average acceleration, velocity, and displacement traces that are specific to each segment. In doing so, strain measurements from the top and bottom of each segment and a boundary displacement are required. Boundary displacement may come from the Statnamic laser reference system (top), top of pile acceleration data, or from embedded toe accelerometer data.

The displacement is calculated at each gage level using the change in recorded strain with respect to an initial time zero using Equation 7. Because a linearly-varying strain distribution is assumed between gage levels, the average strain is used to calculate the elastic shortening in each segment.

Level displacements

$$x_n = x_{n-1} - \Delta\epsilon_{average\ seg\ n} L_{seg\ n} \qquad \text{Equation 7}$$

where

x_n = the displacement at the nth gage level
$\Delta\epsilon_{average\ seg\ n}$ = the average change in strain in segment n
$L_{seg\ n}$ = the length of the nth segment

To perform an unloading point analysis, only the top-of-segment motion needs to be defined. However, the MUP analysis, which is now recommended, requires both top and bottom parameters. The SUP lends itself naturally to providing this information. Therefore, the average segment movement is used rather than the top-of-segment; hence, the SUP actually performs multiple MUP analyses rather than standard UP. The segmental displacement is then determined using the average of the gage level displacements from each end of the segment as shown in the following equation:

$$x_{seg\ n} = \frac{x_{n-1} + x_n}{2} \qquad \text{Equation 8}$$

where $x_{seg\ n}$ is the average displacement consistent with that of the segment centroid.

The velocity and acceleration, as required for MUP, are then determined from the average displacement trace through numerical differentiation using Equations 9 and 10, respectively:

$$v_n = \frac{x_{n_t} - x_{n_{t+1}}}{\Delta t} \tag{Equation 9}$$

$$a_n = \frac{v_{n_t} - v_{n_{t+1}}}{\Delta t} \tag{Equation 10}$$

where v_n = the velocity of segment *n*
a_n = the acceleration of segment *n*
Δt = the time step from time *t* to *t+1*

It should be noted that all measured values of laser displacement, strain, and force are time dependent parameters that are field recorded using high speed data acquisition computers. Hence the time step, Δt, used to calculate velocity and acceleration is a uniform value that can be as small as 0.0002 seconds. Therefore, some consideration should be given when selecting the time step to be used for numerical differentiation.

The average motion parameters (x, v, and a) for segment *m+1* can not be ascertained from measured data, but the displacement at GL^m can be differentiated directly providing the velocity and acceleration. Therefore, the toe segment is evaluated using the standard UP. These segments typically are extremely short (1 - 2 m) producing little to no differential movement along its length.

CALCULATION OF SEGMENTAL STATNAMIC AND DERIVED STATIC FORCES

Each segment in the shaft is subjected to a forcing event which causes movement and reaction forces. This segmental force is calculated by subtracting the force at the top of the segment from the force at the bottom. The difference is due to side friction, inertia, and damping for all segments except the bottom segment. This segment has only one forcing function from GL^m and the side friction is coupled with the tip bearing component. The force on segment *n* is defined as:

$$S_n = A_{(n-1)} E_{(n-1)} \epsilon_{(n-1)} - A_n E_n \epsilon_n \tag{Equation 11}$$

where S_n = the applied segment force from strain measurements
E_n = the composite elastic modulus at level *n*
A_n = the cross sectional area at level *n*
ϵ_n = the measured strain at level *n*

Once the motion and forces are defined along the length of the pile, an unloading point analysis on each segment is conducted. The segment force defined above is now used in place of the Statnamic force in Equation 2. Equation 12 redefines the fundamental equation of motion for a segment analysis:

$$S_n = m_n a_n + c_n v_n + S_{n\,Static} \tag{Equation 12}$$

where, $S_{n\,Static}$ = the derive static response of segment n
m_n = the calculated mass of segment n
c_n = the damping constant of segment n

The damping constant (in Equation 13) and the derived static response (Equation 14) of the segment are computed consistent with standard UP analyses:

$$c_n = \frac{S_n - S_{n\,Static}}{v_n} \qquad \text{Equation 13}$$

$$S_{n\,Static} = S_n - m_n a_n - c_n v_n \qquad \text{Equation 14}$$

Finally the top-of-foundation derived static response can be calculated by summing the derived static response of the individual segments as displayed in the following equation:

$$F_{Static} = \sum_{n=1}^{m+1} S_{n\,Static} \qquad \text{Equation 15}$$

Software capable of performing SUP analyses (SUPERSAW™) is currently being developed at the University of South Florida in cooperation with the Federal Highway Administration.

SITE CHARACTERISTICS AND SUP APPLICATION

Prepared in this section are examples of the motion parameters, segment forces, and load displacement trends as analyzed by SUP. The foundation was instrumented with four strain gage levels (m = 4) which produced five segments. Data was obtained at the 17th Street Causeway Replacement Bridge project as part of an extensive load test program implemented by the Florida Department of Transportation (FDOT), which included Statnamic load tests. Statnamic load testing was performed using a 30MN Statnamic device equipped with a gravel catch structure. Shaft instrumentation consisted of standard Statnamic equipment as well as, resistive type strain gages, and a toe accelerometer. Instrumentation elevations are presented in Table 1.

The test shaft had a planned diameter of 1.22 m and was 22 m in length. It was constructed using a temporary casing method and sea water as the drilling fluid. The 1.22 m O.D. steel casing (1 cm wall thickness) was installed to elevation-18.96 using a vibratory hammer. The concrete was placed using a tremie method, then the casing tip was pulled to elevation -0.9 m, using a vibratory hammer.

A soil boring performed at the test shaft location indicated that the natural ground elevation was approximately 1.5 m. The water table was reported to exist at elevation 0.3 m. SPT testing was initiated at the ground surface (elevation +3 m) and extended to elevation -28.15 m. The upper two meters of soil consisted of compacted limestone fill with SPT “N” values ranging from 27 to 16.

Table 1 Instrumentation Schedule

Instrumentation Elevation (m)	Number of Transducers	Type of Transducer
3.0	4	Calibrated Load Cell, 2 Accelerometers, and Laser Reference System
-1.8	3	Strain Gage
-4.2	3	Strain Gage
-17.0	3	Strain Gage
-18.3	3	Strain Gage
-19.0	1	Accelerometer

The following strata was reported as fine sand with fragments of limestone and shell. This strata extended to elevation -14.7m, "N" values ranged from 9 to 57. From elevation -14.7m throughout, the rock socket length averaged 34% RQD at 80% recovery. RQD values ranged from 18% to 73% in the limestone below the shaft tip. Recovery values in this strata are generally greater than 70%.

Figure 5 shows the measured change in strain with respect to time $t = 0$ for each gage level. Figures 6 through 8 illustrate the motion parameters determined for each of the five segments. Figure 9 shows the forces calculated at each gage level with the true measured strain. Figure 10 shows the dynamic forces on each of the segments as calculated by Equation 11.

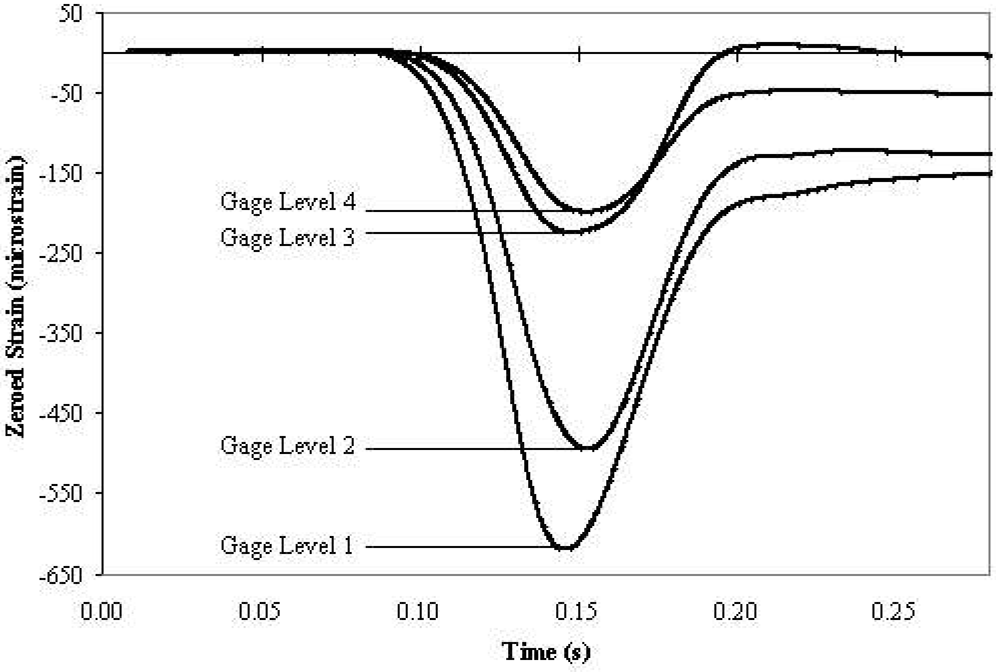

Figure 5 Strain versus Time

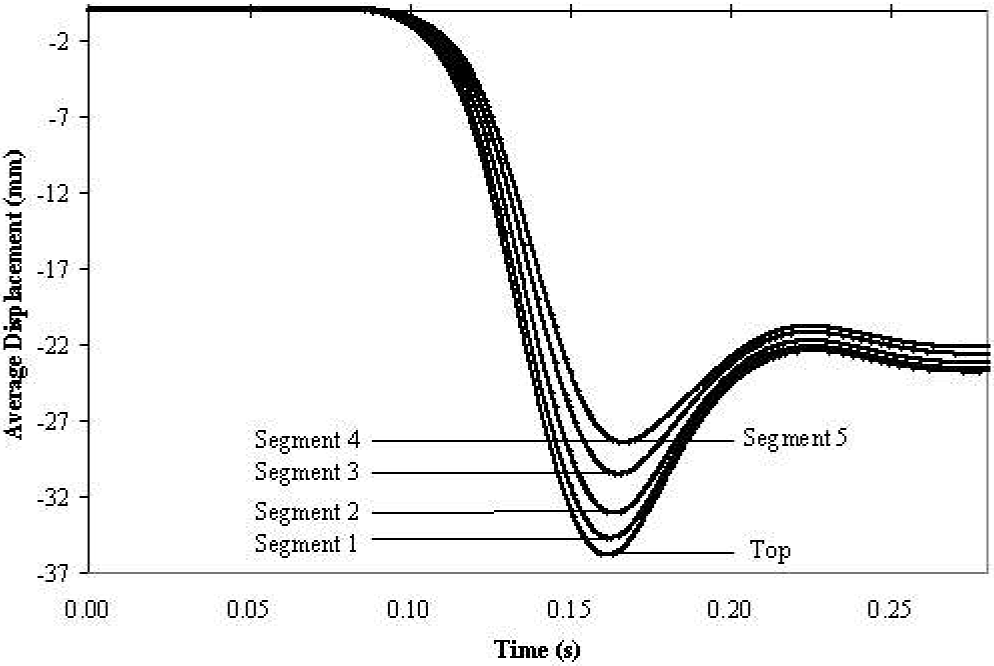

Figure 6 Segmental Average Displacements

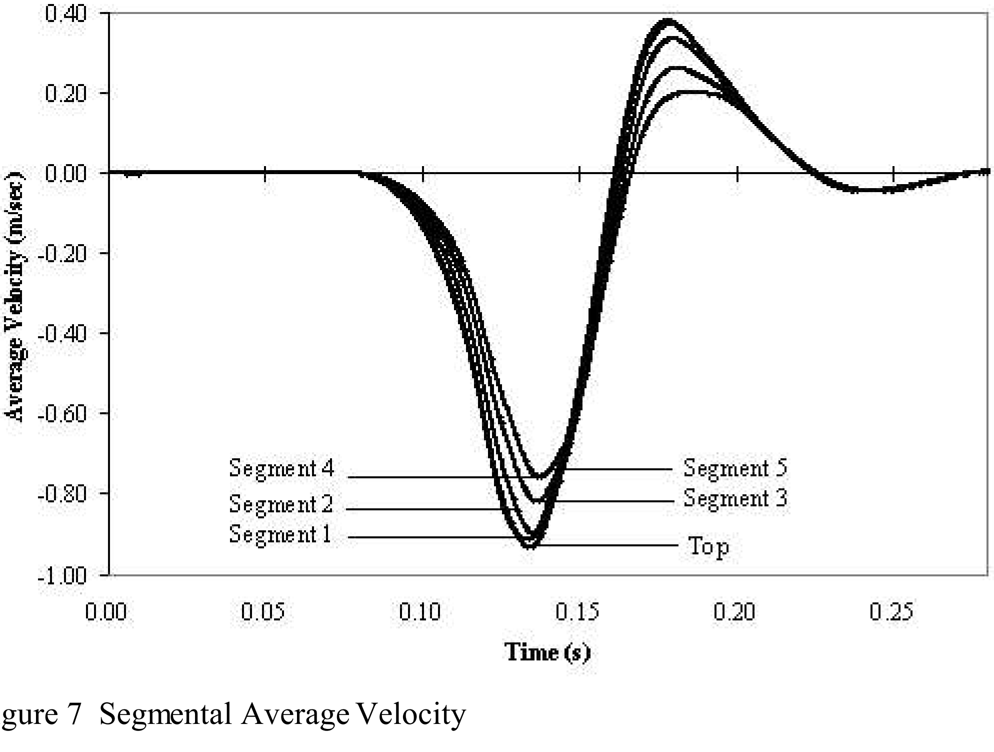

Figure 7 Segmental Average Velocity

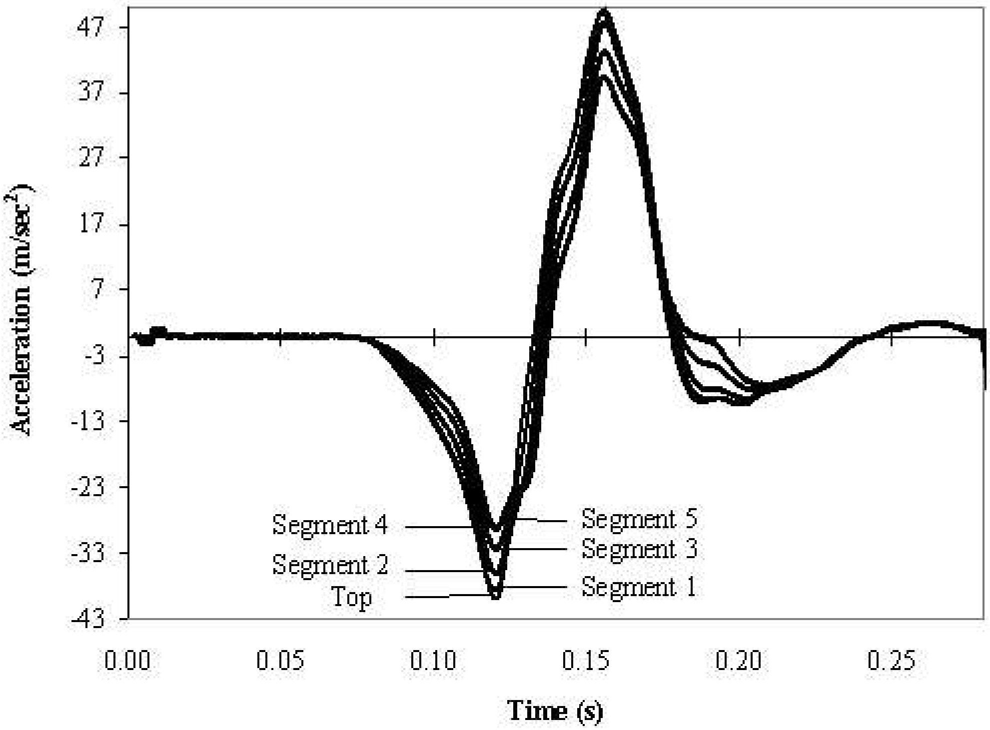

Figure 9 Segmental Average Acceleration

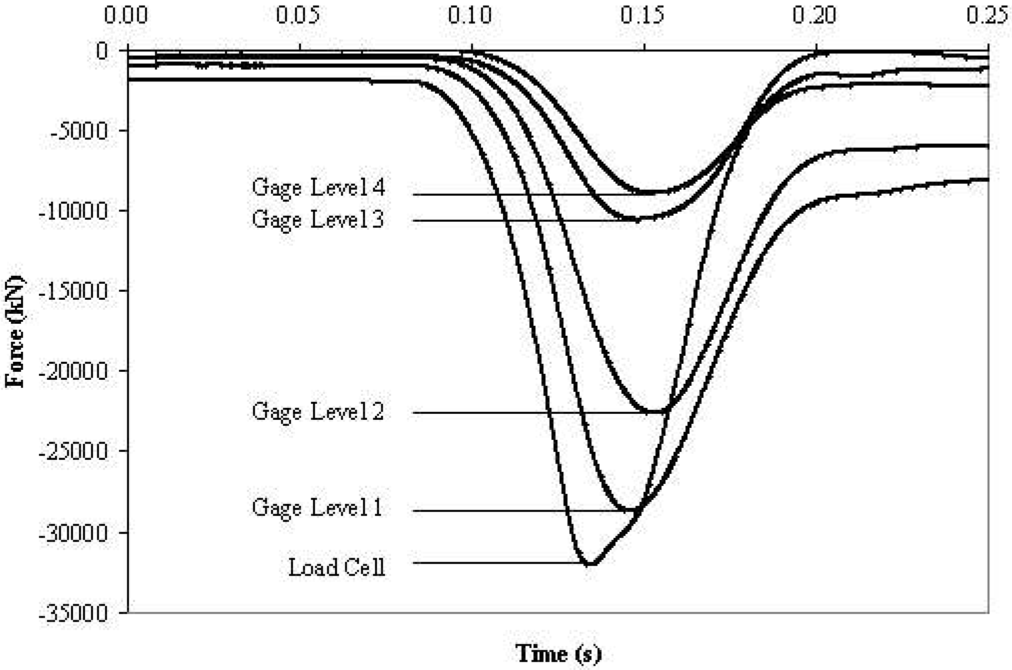

Figure 8 Force at Gage Levels

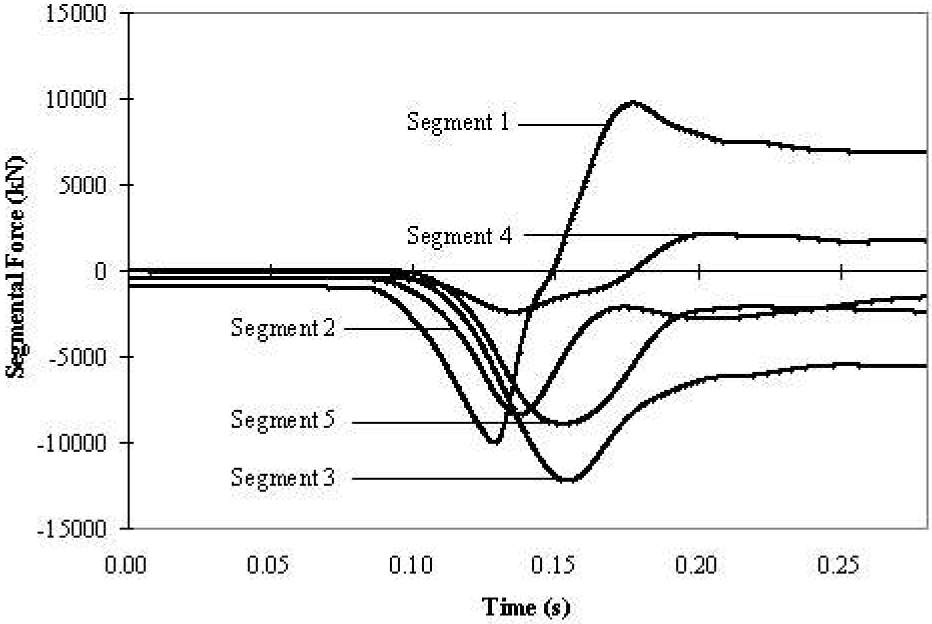

Figure 10 Segmental Statnamic Forces

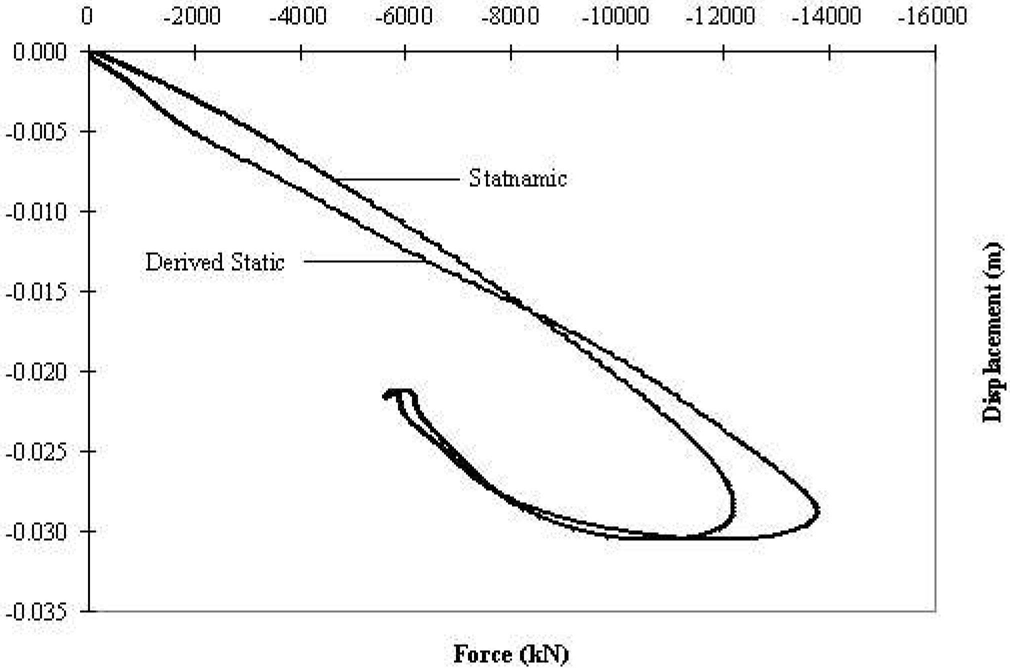

Figure 11 Segment Load Displacement Curves

Using the segmental forces, the derived static soil resistance is determined for each of the segments. Figure 11 shows a typical segment load versus segment displacement curve. By simply dividing the segment force by circumferential surface area this curve can be converted in to a shear stress versus displacement (T-Z) curve for that specific soil strata.

It can be seen in Figure 10 that the peak of each segment force may occur at different times and therefore at different top-of-pile displacements. This can effect the pile capacity in that the ultimate shear strengths of the strata should not be simply summed. This is most probably not a significant concern with this pile due to the plunging nature of the failure. This is evidenced by the similar top and toe movements shown in Figure 6. However, to be technically correct, SUP uses the summation of segment forces as they were developed. This accounts for upper soil layers that may fully mobilize and become residual in nature while lower soil layers begin to develop ultimate strengths. Figure 12 shows the raw Statnamic load displacement curve as well as SUP, MUP, and UP derived static capacity versus the top-of-pile displacement.

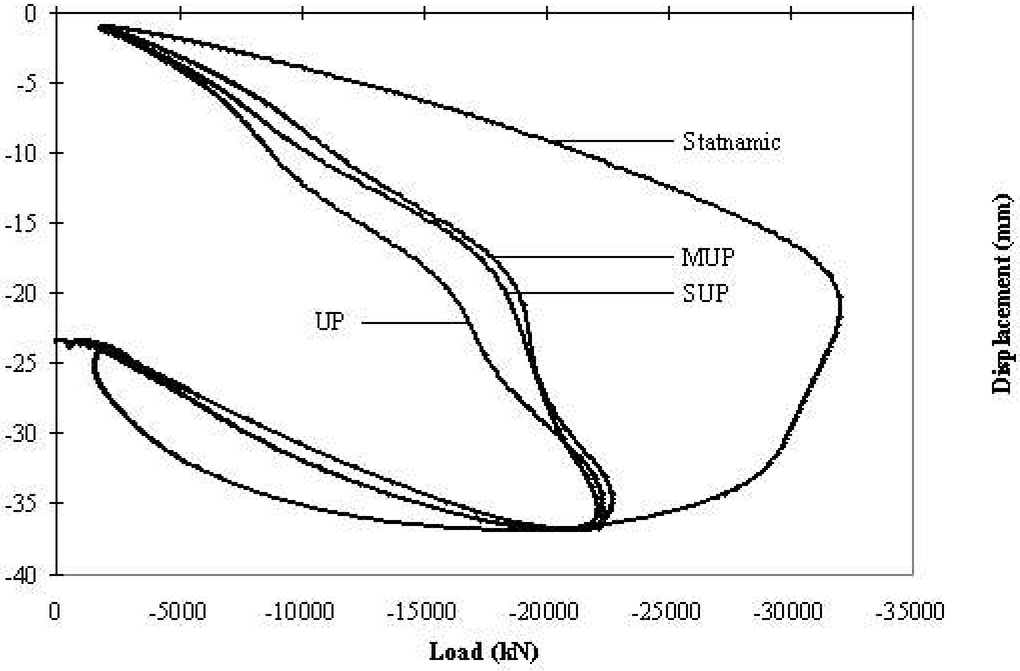

Figure 12 Top of Pile Load versus Displacement

SUMMARY

A new method of analysis called the Segmental Unloading Point Method (SUP) was presented that evaluates Statnamically loaded foundations as segments whose lengths are defined by embedded strain gage elevations. The recorded strain measurements are used to determine both the segmental motion parameters as well as the segmental force traces. Each segment is then treated as an individual foundation whose static response is derived using either the UP or MUP methods. The summation of each segment contribution with respect to time provides a top of foundation response that more closely incorporates the actual distribution of inertial and damping forces throughout the foundation. This is most important in the analysis of relatively long or fixed-ended piles. Although the UP and MUP methods of analysis are sufficient for most loading conditions, SUP provides information for soil strata T-Z curves as well as cut-off elevation load-displacement curves.

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